LABORATORI D'ENVOL
Glider Engineering

LEPARAGLIDING 3.20V

NOTES AND CHANGES OVER PREVIOUS LEP VERSIONS

This version is the final 3.20 version. Developed in August and September 2022

Acknowledgments:

Report version 8

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Teià, 2022-09-20

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1. OVERVIEW

Leparagliding-3.20V adds five new sections to the leparagliding.txt data file over the previous version 3.19. Also, there are small changes to three existing data sections. However, you don't need to worry too much! A file in 3.20V format will also work in program version 3.19, and some earlier, without any change.

The three sections with changes are:
- Section 8 Global angle of attack estimation
- Section 9 Suspensions lines description
- Section 10 Brakes

The five new sections are:
- Section 33 Detailed risers
- Section 34 Lines characteristics table
- Section 35 Solve equilibrium equations
- Section 36 Create files for XFLR5 analysis
- Section 37 Some special parameters

Likewise two new folders stl/ and xflr5/ are automatically generated at the same level as lep/

With these changes it is possible to automatically or explicitly define the diameters and properties of each of the lines.

New .xwimp geometry file and .dat profiles are automatically generated to be imported into the XFLR5 program and perform aerodynamic studies. This complements the .stl files generated in the previous version that can be used to analyze the wing surfaces with CFD programs.

Now, the program performs geometric and physical calculations internally, to calculate all the forces and angles involved and try to solve the longitudinal balance equations. There is a very detailed calculation of the aerodynamic drags and weights of each one of the lines. The drag is calculated for each line taking into account its surface and the drag coefficient adapted to the Reynolds number with which each one works, flow speed and air parameters, and taking into account the effect of the loops.

In the lines.txt file, the total lengths of each type of line according to diameters and materials have been added, to facilitate the work of the constructors.

In the lep-out.txt output file, new sections 15,16,17,18. 19. have been added with information about the new sections. Particularly interesting is section 17 with abundant geometrical and physical data on the paraglider. Presentation of lep-out.txt slightly improved. Display with fixed-width font.

We will soon apply the 3.20V program to the new gnuComet and gnuReflex paragliders, currently in the design phase.

2. SIMPLIFIED NEW SECTIONS STRUCTURE

In LEP-3.20V you don't need to complicate yourself with the new sections. Add this code below your leparagliding.txt file, and everything will continue as before!

```
*******************************************************
*       33. DETAILED RISERS
*******************************************************
0
*******************************************************
*       34. LINES CHARACTERISTICS TABLE
*******************************************************
0
*******************************************************
*       35. SOLVE EQUILIBRIUM EQUATIONS
*******************************************************
0
*******************************************************
*       36. CREATE FILES FOR XFLR5 ANALYSIS
*******************************************************
0
*******************************************************
*       37. SOME SPECIAL PARAMETERS
*******************************************************
0
```
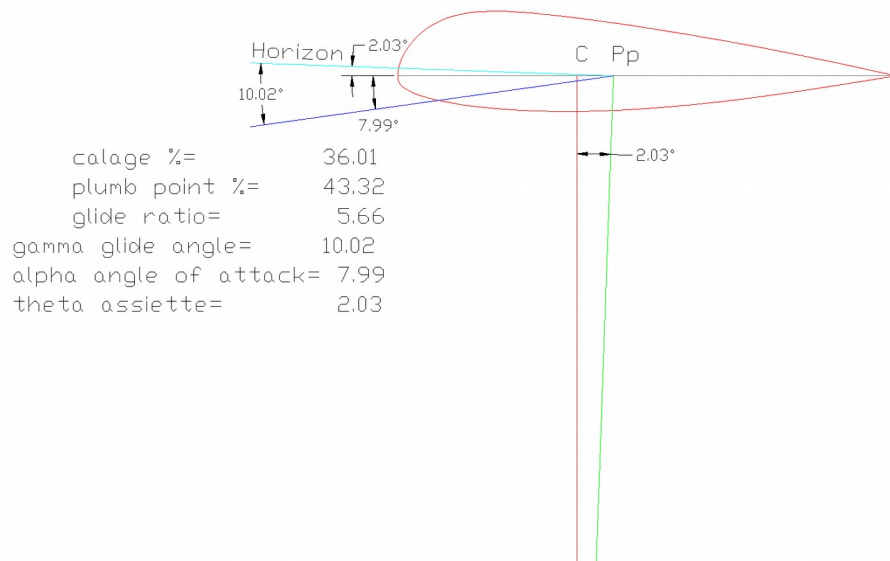
Therefore, if you want it is not necessary to continue reading! :)

3. SECTION 8: GLOBAL ANGLE OF ATTACK ESTIMATION

This section should be completed once a complete analysis has been performed according to SECTION 35. However, you can continue to use the traditional method and remember that the really important parameter is the "CALAGE", which is the position of the pilot with respect to the wing and that will completely define all flight characteristics. Normally, we determine the ideal calage based on experience with similar previous models, empirical formulas, and flight tests using trimmers.

From this version (3.20V), the parameter we previously called "Cp" (pressure center), we will now call it "Pp" (plumb point, the point on the central chord, located just at the same vertical of the pilot), since this definition is more appropriate to physical real world. The center of pressure is another point, to be determined precisely by numerical aerodynamic analysis.

```
************************************************************
*            8. Global angle of attack estimation        *
************************************************************
* Finesse GR
      5.66
* Plumb point estimation %
      43.32
* Calage %
      36.01
* Risers lenght cm
      47
* Line lenght cm
      620
* Karabiners cm
      40
```



```
                 Horizon ┌2.03°                    C  Pp
              10.02°
                               7.99°

       calage %=              36.01
       plumb point %=         43.32
       glide ratio=            5.66
    gamma glide angle=        10.02
    alpha angle of attack=    7.99
    theta assiette=            2.03
```

The finesse, plumb point, and calage parameters can be defined before or after performing an optional aerodynamic analysis according to section 35.

4. SECTION 9: SUSPENSION LINES DESCRIPTION

Optionally, (is not mandatory) now is possible to describe the type, diameter, materials and other characteristics of each individual line. You need to add some parameters (integers) designate "type" according to the table in section 34, or the table predefined internally in the program by default.

This will serve to force the definition of line types and thicknesses in the design phase, automatically calculate the required lengths of each type of line, and calculate aerodynamic parameters (drag and weight for static balance effects, according to section 35).

Example:

```
***************************************************
*          9. SUSPENSION LINES DESCRIPTION
***************************************************
3
3
13
3    1 1   2 1   3 1   0 0   1 1   -  1 2 4
3    1 1   2 1   3 2   0 0   1 2   -  1 2 4
3    1 1   2 1   3 3   0 0   1 3   -  1 2 4
3    1 1   2 2   3 4   0 0   1 4   -  1 2 4
(…)
```

The integer numbers added after the character "-" mean:
1 --> in level 1 use line type "1"
2 --> in level 2 use line type "2"
4 --> in level 3 use line type "4"

If you do not specify the type of each line for each level, the program internally specifies the "i" type at "i" level from i 1 through 4, except for the main brake line 1F1 which is automatically assigned as type 6. It is necessary to assign a type to all the lines, or to none of them (it is not allowed to assign types only in some lines).

5. SECTION 10: BRAKES

Same description types as section 9.

6. SECTION 33: DETAILED RISERS

Type a single parameter "0" to bypass this section and use pre-set values :)
Type 1 and additional parameters to design a paraglider with risers of different lengths A, B, C, D, E (not usual, but sometimes it may be necessary). Works.

Type 2,3 or 4 to project the risers of a vario seat system of two, three, four points (types 2,3,4 still not available).

```
********************************************************
*        33. DETAILED RISERS
********************************************************
1
1
A      45.0   cm
B      50.0   cm
C      60.0   cm
```

7. SECTION 34: LINES CHARACTERISTICS TABLE

Type a single parameter "0" to bypass this section and use predefined typical values.

Definition table of the properties of N different types of lines used in our paraglider. Up to 50 different types of lines.
Each type is described in a line with 12 positions:
1 --> Line type
2 --> r or c (r=rectangular axb or c=circular section)
3 --> line diameter (mm) or a dimensions (mm)
3.5 --> nothing or b dimension (mm), used only in "r" types
4 --> line label "Riser", "PPSL275", "DC60",... use names without spaces up to 15 characters
5 --> minimum breaking strenght (daN)
6 --> daN
7 --> material type "dyneema", "aramid", "polyester"... use names without spaces up to 15 characters
8 --> weight per line meter (g)
9 --> g
10 --> s or p (s=sewed or p=spliced loop)
11 --> total loop length (cm)
12 --> cm
13 → set line CAD color, used only if code 1341 is active in section 37 (idea by Eric Fontaine)

Define your own lines types according to manufacturers' tables. Example:

```
********************************************************
*        34. LINES CHARACTERISTICS TABLE
********************************************************
1
6
1  r  25.  2. Riser     1000 daN  polyester   20.0   g   s 12. cm  7
2  c  1.90    Line275     275 daN  s_dyneem     2.26   g   s 12. cm  1
3  c  1.40    Line160     160 daN  s_dyneem     1.34   g   s 10. cm  3
4  c  1.15    Line120     120 daN  s_dyneem     1.00   g   s 10. cm  5
5  c  0.80    Line100U    100 daN  u_dyneem     0.43   g   p 8.  cm  2
6  c  2.00    Line200B    200 daN  s_dynemm     3.10   g   s 12. cm  6
```

8. SECTION 35: SOLVE EQUILIBRIUM EQUATIONS

Type a single parameter "0" to bypass this complicated section! :)

Definition of the initial basic parameters used to solve the longitudinal equilibrium of the paraglider. This section is informative and is used by the designer, to study the values of the forces involved in the balance of the wing, the flight speed, the angles, and the glide coefficient.
To find realistic values, it is necessary to do the study simultaneously with the XFLR5 program or CFD programs, and perform several iterations until satisfactory values are obtained. Currently, it is not yet possible to fully automate this calculation. The designer must apply his criteria according to the type of wing under study.
We have discussed this section extensively with Francois de Villiers during the last few weeks, using different approaches to the final solution.

```
********************************************************
*       35. SOLVE EQUILIBRIUM EQUATIONS
********************************************************
1
g        9.807  m/s2 gravity of Eart
ro       1.225  kg/m3 air mass density
mu       18.46  muPa·s air dynamic viscosity (microPascals)
V        12.7   m/s  estimated flow speed
Alpha    9.45    deg  estimated wing angle of attact at trim speed
Cl       0.55619     wing lift coefficient
cle      1.0         lift correction coefficient
Cd       0.03560     wing drag coefficient
cde      1.35        drag correction coefficient
Cm       0.0         wing moment coefficient
Spilot   0.438  m2   pilot+harness frontal surface
Cdpilot  0.6         pilot+harness drag coefficient
Mw       5.0    kg   wing mass
Mp       65.9   kg   pilot mass included harness and instruments
Pmc      0.2    m    pilot mass center below main karabiners
Mql      8.0    g    one quick link mass (riser-lines)
Ycp      0.489  m   y-coordinate center of pressure
Zcp      0.299  m   z-coordinate center of pressure
```

Explanations:

g → gravity of Earth (9.80665 m/s2 standard gravity)
ro → air mas density kg/m3
mu → air dynamic viscosity microPascals·s
V → estimated initial flow speed m/s, used for first Cl, Cd, Cm values
Alpha → estimated ideal angle of attack deg. Max glide ratio according wing aerodynamic analysis
Cl → Wing lift coefficient, obtained by analysis with individual profiles, XFLR5, or CFD
Cle → multiplier coefficient of Cl, to consider non-modeled geometries, use 1.0 in case of doubt
Cd → Wing drag coefficient, obtained by analysis with individual profiles, XFLR5, or CFD
Cde → multiplier coefficient of Cd, to consider non-modeled geometries, use 1.15 in case of doubt. If the Cd data comes from CFD this coefficient can be very close to 1.0. Currently studying how this

coefficient affects the results. Probably by adjusting through Cde the expected GR, the rest of the parameters will be very close to reality.

Cm → Wing moment coefficient, obtained by analysis with individual profiles XFLR5, or CFD

Spilot → Pilot + harness frontal surface (m2)

Cdpilot → Pilot+harness drag coefficient (depends on the type of harness, especially if have fairings)

Mw → Wing mass (kg) without lines and risers

Mp → Pilot+harness+instruments mass (kg)

Pmc → Pilot+harness mass center distance from main carabiners (m)

Mql → Mass of one quicklink used to connect riser with lines (kg)

Ycp → Y-coordinate of center of pressure (m), obtained by analysis with individual profiles, XFLR5, or CFD

Zcp → Z-coordinate of center of pressure (m), obtained by analysis with individual profiles, XFLR5, or CFD

Remember that the axes used in LEparagliding are:
 Origin (0,0,0)= at the nose of the central profile section.
 X-axis horizontal and in the span direction
 Y-axis along the central chord
 Z-axis perpendicular to the XY plane and pointing down (not coincides with gravity axis)

9. SECTION 36: CREATE FILES FOR XFLR5 ANALYSIS

0 --> don't perform xflr5 analysis
1 --> set parameters for xflr5
* Panel parameters
10 chord nr
5 per cell
1 cosine distribution along chord
1 uniform along span
* Include billowed airfoils (more accuracy) [Still not working]
0

If you use this section, it will automatically be created in xflr5/ directory with a .xwimp file and profiles in .dat format to use in an aerodynamic analysis with the XFLR5 program. The details of how to do it are explained here: http://www.laboratoridenvol.com/info/lep2xflr5/lep2xflr5.html

Unfortunately with XFLR5 we cannot model paragliders with profile rotations in the Z angle, nor single skin paragliders. CFD programs must be used for this type of paraglider.

10. SECTION 37: SOME SPECIAL PARAMETERS

This section will be a "wildcard", to add special parameters to control things from previous or future data sections… It is an unorthodox and somewhat "dangerous" section, because any parameter or modification of previous or future ones could be added here. (!).
It will consist of an initial control parameter "0" to end and define nothing! :)
Or the parameter "1" to activate the section, and below the parameter N which means add N lines with special parameters.
Each line will start with a control code, an integer that will have a meaning to be specified. And then, on the same line, one or more parameters (integers, reals, or characters) related to the subject of the control code.
It may seem a little abstract..., but you will see that it will be very practical...
For example, it can be used to activate transitions in the thickness of miniribs (HEGALA style...), or add nylons in the middle of the inlets...

The codes available in 2022-09-18 are:

- **Code 1146**, which means that it is necessary add a real estimated mass center % of a typical section (0% leading edge, 100% trailing edge). Used only to calculate the estimated wing center of mass. Default value is 36%.
- **Code 1291**, which means that it is necessary to write on the right an integer number which is the number of transversal segments to use on the STL surfaces (related to section 29)
- **Code 1341**, if code is followed by a "1", then the lines will be drawn according to the colors by type defined in section 34. If it is a "0" the colors will be drawn according to the risers, in accordance with sections 24 and 25 . If code 1341 does not exist, the colors are defined by sections 24 and 25.
And some "*Programmer secret parameters*" (maybe, they will be removed soon):
- **Code 1351**, set the solve method of equilibrium equations. Six different methods are currently being calibrated.
Method 1: The Lift and Drag are multiplied by a coefficient K to homogenize their value. The system of vertical (I) and horizontal (II) equilibrium equations is solved numerically for the unknowns K and *gamma* (glide angle), and all other parameters being data. A continuous range of gamma values is explored, and when the K1 values in the vertical equation match the K2 value in the horizontal equation, the system is considered solved. By adding to the data file some additional drag coefficient Cde=1.1 to 1.4 the results are quite realistic. This extra drag coefficient tries to represent the geometry not modeled in the aerodynamic calculation (probably billow and inlets).
Method 2: In the balance equations an Extra Drag of the wing is considered: The two unknowns to be solved are gamma and Dragextra, with a numerical method similar to the previous case, we explore a range of gamma until the Dragextra values match in the two equations . With some initial values of Lift and Drag, the system does not converge, a warning message is displayed in output file, and then general scaling adjustments in pilot weight or wing area are required.
Method 3: Combination of the previous two, first a coefficient K is provisionally calculated, to homogenize the Lift and Drag, and then the two equations (V,H) system is solved by gamma angle and extra drag. This method provides quite good results.

Method 4: Solves glide angle *gamma* directly using analytical expression (horizontal equilibrium). Use code 1352 set to 2 when using method 4. Flight speed is obtained from the Lift equation (same as in the previous methods).

Method 5: Solves glide angle *gamma* directly using analytical expression (horizontal equilibrium), as in method 4. Then solves flight speed numerically, from the vertical equilibrium equation. Then computes again gamma, Lift, Drag and other values.

Method 6: This is a fully analytical method. The glide angle calculation *gamma* is based on the horizontal balance equation. *Gamma* is expressed from initial values (and results of intermediate calculations), and independent of flight speed. The flight speed is also calculated analytically from the vertical balance equation. With the flight speed calculated, the Lift and Drag values are calculated again, and used in the moment balance equation to obtain the ideal trim "calage". Also other values derived in simple form. Thus the system is completely resolved in a very elegant way.

The calage calculation is carried out using only the moment balance equation (III), once the *gamma* (glide) and *theta* (assiette) angles are known, and is the same for all methods.

The accuracy of the methods and their applicability is currently under study. **It is recommended to use only method 1 or method 6**. The results are practically the same. Results of methods 1 and 6 are consistent with the experimental results obtained with Hegala and Tuluaq paragliders. The parameter "cle" (extra lift coefficient) is usually set to 1.0. The parameter "cde" (extra drag coefficient), has a key importance in the final result. It represents the drag that the numerical model used (XFLR5 o CFD) could not determine. For practical purposes it is necessary to use "cde" values based on previous paraglider designs, or simply adjust cde until the output glide ratio is as expected... This will provide a calage value close to reality.

If code 1351 is not specified, the default used method is 1. Methods 2 to 5 are still under review, do not use.

- **Code 1352**, set 1 to use flat area, or 2 to use projected area, in Lift, Drag, and speed calculus. Due to the internal adjustments made, this aspect has almost no influence on the results. By default the value used is 2. Use value 2 when using method 4 in code 1352. It is a code that the regular user does not need to specify

- **Code 1353**, set 0 to output the standard report in section 17 of lep-out.txt, or set to 1 or 2 for more detailed output used for studies and calibrating methods. Default is 0. Most verbose mode is 2.

Example:

```
*****************************************************
*       37. SOME SPECIAL PARAMETERS
*****************************************************
1
6
1291   12   nsegments in transverse direction (stl surfaces)
1341   1    use CAD colors for each line type according table 34
1146   36.  gravity center in % of a typical section (default is 36%)
1351   1    Solve method (1 or 6), default is 1
1352   2    Use flat area (1) or projected area (2), default is 2
1353   2    Print normal (0) or advanced detailed output (1),(2)
```

Type a single parameter "0" to bypass this section! :)

11. TWO NEW DIRECTORIES NECESSARY AL THE SAME LEVEL AS LEP

Folder: stl/
All .stl and .scad files are now grouped in this folder, in order not to accumulate an excess of files in the lep directory.

Folder xflr5/
New automatically generated geometry file ".xwimp" and all ".dat" necessary airfoils are put in this folder, used to import directly into the xflr5 external program and perform numerical analyzes.
Soon 1 or more billoweb airfoils will be added automatically to each cell, to make a more precise analysis.
This section made possible by the studies done by Francois de Villiers, on how to export LEP geometry to XFLR5 and how to choose the right parameters.

12. OUTPUT FILES

Output file lines.txt:

Added the type and diameter specified for each line.
Automatic calculation of total length of lines with loops and without loops.
Table of required lengths for each type of line. including loops, to facilitate the selection of materials.
The selection of line types works automatically, without the need to specify anything, but it is fully configurable.

Output file lep-out.txt:

Small improvements in the general presentation.
Added wing estimated mass center in section 1.
Detailed individual lines and risers properties (lenght with and without loops), mass, area, and center of mass. Global lines and risers properties (total mass (kg), area (m2), mass center coordinates). Added some output for new sections 33,34,35,36,37.

Section 17. of lep-out.txt:

With this version of LEparagliding, for the first time an automatic calculation of the longitudinal balance is carried out. It is necessary to provide first the basic aerodynamic parameters of the wing (without lines or pilot) coefficients Cl, Cd, Cm, and point Cp, obtained with an external program. The rest of the geometric and aerodynamic parameters are generated internally by Leparagliding.

The main results are the glide angle *gamma*, the angle of the wing relative to the horizon (*theta* or "assiette"), and recommended position of the pilot with respect to the wing central chord "calage". It

may be necessary to perform several iterations with different initial speeds and angles of attack until values converge to real values.

This is an advanced section that should be used with caution. However, the geometric calculation of the wing is still done according to section 8. Therefore, the designer will decide whether or not to use the provided values of calage, glide ratio, and plumb point, to manually insert into the section 8. To stabilize the results provided in section 17 of lep-out.txt, run the program at least one or two additional times, updating the parameters in section 8 of leparagliding.txt until convergence.

```
 An attempt to solve the paraglider longitudinal equilibrium. Use interactively
until you get convincing results.
 It is necessary to use external programs such as XFLR5 and or CFD to obtain the
aerodynamic coefficients and the center of pressure
 Remember that the axes used in LEparagliding are:
 Origin (0,0,0)= at the nose of the central profile section.
 X-axis horizontal and in the span direction
 Y-axis along the central chord
 Z-axis perpendicular to the XY plane and pointing down (not coincides with gravity
axis)

 Basic project data:
 -------------------------------------------------
     Flat area=   26.20 m2  282.0 ft2
     Flat span=   10.57 m    34.7 ft
      Flat A/R=    4.27
Projected area=   22.81 m2  245.5 ft2
Projected span=    8.66 m    28.4 ft
 Projected A/R=    3.29
    Flattening=   12.94
   Vault arrow=    2.48 m
Proj_span/arrow=   3.49
   Line heigth=    6.98 m included risers
Proj_span/Line_heigth=    1.33
Karabiners - wingtip=    5.92 m
Proj_span/(Karabiners - wingtip)=    1.46
  Wing type is= ds
Planform geometric center of gravity at  45.37 % from leading edge 1535 mm

 Section 35 data and initial parameters:
 -------------------------------------------------
              g=    9.8070 m/s2 gravity of Eart
             ro=    1.2250 kg/m3 air mass density
             mu=   18.4600 microPascals x s air dynamic viscosity
              V=   11.0000 m/s estimated initial flow speed
          Alpha=    9.0000 deg estimated wing angle of attack AOA
             Cl=    0.5800 wing lift coefficient
            Cle=    1.0000 wing lift adjustment coefficient
             Cd=    0.0300 wing drag coefficient
            Cde=    1.2000 wing drag adjustment coefficient
             Cm=    0.0000 wing moment coefficient
            Cdp=    0.8000 pilot drag coefficient
         Spilot=    0.5000 m2
      Wing mass=    4.000 kg
     Pilot mass=   83.000 kg included harness, instruments, water...
Pilot mass desp=    0.000 m  below main karabiners
```

```
x-coordinate Cp=      0.000 m
y-coordinate Cp=      0.930 m
z-coordinate Cp=      0.250 m

 Results:
 ------------------------------------------------
   Solve method=       1

 Masses and wingload:
    Lines mass=       0.767 kg (risers included)
    Risers mass=      0.091 kg
   Quickli mass=      0.048 kg    6 units
    Total mass=      87.815 kg
    Wing flatS=      26.2019 m2
    Wing projS=      22.8123 m2
      Wingload=       3.351 kg/m2 (using flat wing area)
      Wingload=       3.849 kg/m2 (using projected wing area)

 Lines calculus:
    Lines surf=       0.7200 m2 (risers included)
    Risers surf=      0.0775 m2
   y-Lines drag=      1.3973 m
   z-Lines drag=      3.5114 m
Global Cdlines=       1.1876

 Aerodynamic forces:
      k1,k2,k=        0.8656      0.8656      0.8656
     Wing lift=   848.7830 N
  Wing lift ex=     0.0000 N
 Wing lift tot=   848.7830 N
     Wing drag=    43.9026 N
  Wing drag ex=     8.7805 N
 Wing drag tot=    52.6831 N
    Lines drag=    63.3724 N (risers included)
    Pilot drag=    29.6450 N
    Total drag=   145.7005 N
  Induced drag=    46.8696 N (theoretical by AR, and included in wing drag)

 Lift to Drag ratios:
    Wing Cl/Cd=    19.3333
Cl*Cle/(Cd*Cde)=    16.1111
  L/(D+Dextra)=    16.1111
 L/(Drag total)=    5.8255

 Theoric speed=    11.000 m/s      39.60 km/h
   Horiz speed=    10.841 m/s      39.03 km/h
    Vert speed=     1.861 m/s       6.70 km/h

 Mass centers:
      Wing mc:
            y=      1.184 m
            z=      0.712 m
Lines+risers mc:
            y=      1.392 m
            z=      3.429 m
  Quicklinks mc:
            y=      1.276 m
```

```
             z=        6.488 m
Pilot+harnes mc:
             y=        1.327 m
             z=        6.980 m
      Total mc:
             y=        1.321 m
             z=        6.663 m

Angles and calage:
WARNING: Iterate initial values until convergence!
These results are for informational purposes only!
The REAL design parameters must be entered manually in section 8 of
leparagliding.txt
          Theta=       0.740 deg
          Gamma=       9.740 deg
    Glide ratio=       5.826
 Main-K x-coord=       0.200 m
 Main-K y-coord=       1.327 m
 Main-K z-coord=       6.980 m
           Cp %=      27.49 %    930 mm
           Pp %=      41.88 %   1416 mm
        Calage %=      39.21 %   1326 mm (Py)

 Complements:
        Pp-Py %=        3.45 %     90 mm
        Py-Cp %=       15.16 %    396 mm
        Pp-Cp %=       18.61 %    486 mm
      wcg-wcp=  527 mm Distance(wingcg(y,z),wingcp(y,z))
   wcg_y-wcp_y=  254 mm
   wcg_z-wcp_z=  461 mm

 Detailed lines drag information:
 --------------------------------------------------
 Line - Surf (mm2) - Reynolds -  Cdline  -  Line drag (N)
   1      11250.       18249.       1.980        1.65
   2       9117.        1387.       1.080        0.73
   3       9022.        1387.       1.080        0.72
   4       8852.        1387.       1.080        0.71
   5       8491.        1387.       1.080        0.68
  (...)
```

The content of the report is self-explanatory. And as you can see with very interesting geometric and physical data. Ask if you do not understand the meaning of any parameter.

The mysterious parameters k1,k2,k do require explanation (used only in methods 1 and 3). Parameter K1 is a coefficient multiplying the absolute value of Lift and Drag in the equation of vertical balance of forces. Parameter K2 is a coefficient multiplying the absolute value of Lift and Drag in the equation of horizontal balance of forces. The simultaneous solution of the vertical and horizontal balance equations is performed numerically by exploring a range of glide angles (gamma) between 1º and 45º. In the vertical balance equation the coefficient K1 is calculated as a function of gamma. In the horizontal balance equation, the K2 coefficient is calculated as a function of gamma. When the coefficients K1 and K2 are equal or almost equal, then K=(K1+K2)/2 is defined. This is considered the solution, the pair of unknowns (gamma,k), of the vertical and horizontal equilibrium equations. That is, in solving the vertical and horizontal balance equations, we are forcing the total wing lift and wing drag to be

multiplied by the reducing or amplifying coefficient K. This strategy is an unproven conjecture, which provides consistency between the absolute values of Lift and Drag with the rest of the aerodynamic and gravitational forces.
Then total wing lift is:
Lift=(1/2)*Cl*Cle*K*ro*V^2*S
And total wing drag is:
Drag=(1/2)*Cd*Cde*K*ro*V^2*S
Where S is the flat wing surface (m2).
Note that the multiplication by S is somewhat arbitrary. That is why it is necessary to add the K coefficient that regulates Lift and Drag in order to be compatible with the independent aerodynamic forces (drag of lines and pilot) and of gravity (weight of the wing, weight of lines, weight of the pilot).

In Appendix 2 you can see the detail of the balance equations, and how the coefficients K1,K2,K contribute.

About the theoretical flight speed. In methods 1 to 5, we consider the theoretical flight speed according to the simplified formula: V=sqrt((W*cos(gamma))/((1/2)*ro*Cl*Cle*K*S))
where W is the total weight and W*cos(gamma) means that the vertical component of the Lift equals the total weight. This formula is not exact and is not independent of the rest of the calculations made. In method 6 the flight speed is obtained analytically from vertical equilibrium equation.

The most suitable would be to do a study for several initial flight speeds (V=9,…,12 m/s) around the one estimated initially. Examine the variations in glide ratio and calage values obtained, and then decide accordingly.

13. SOLVING THE EQUILIBRIUM EQUATIONS

To solve a system with N unknowns, we need at least N independent equations or conditions. Consider the simplified diagram of a paraglider in balanced and stable flight, and consider all the parameters involved.

<u>Unknowns we have:</u>

Flow speed:

[1] V flight speed (m/s)

Coefficients:

[2] Cl wing lift coefficient
[3] Cle wing extra lift coefficient
[4] Cd wing drag coefficient
[5] Cde wing extra drag coefficient
[6] Cm wing moment coefficient
[7] K multiplier coefficient for lift and drag

Some drags:

[8] Dlines   lines drag (N)
[9] Dpilot pilot drag (N)

Angles:

[10] alpha wing angle of attack (deg)
[11] gamma glide angle (deg)
[12] theta *assiette* inclination of the central chord above or below the horizon (deg)
[13] GR glide ratio

Points:

[14] Xcp y-coordinate center of pressure (m)
[15] Zcp z-coordinate center of pressure (m)
[16] C *calage*, where to place the pilot with respect to the central chord (m or %)
[17] PP plumb point, intersection of vertical line by the pilot, with the central chord (m or %)
[18] Gy wing mass y-coordinate (m)
[19] Gz wing mass z-coordinate (m)
[20] Gly y-coordinate of lines mass (m)
[21] Glz z-coordinate of lines mass (m)
[22] GDly y-coordinate of lines mass (m)
[23] GDlz z-coordinate of lines mass (m)

Weights:

[24] Mpilot pilot mass (kg)
[25] Mw wing mass (kg)
[26] Mlines lines mass (kg)
[27] Mquicklinks quick links (maillons) mass (Kg)
[28] Mtotal (kg)

Geometry:

[29] h the distance from pilot to central chord (in a perpendicular line) (m)

A bit exaggerating, we have about 29 unknowns... We need at least 29 independent conditions to solve the system.

Conditions to solve the system:

[1] V flight speed (m/s) → Set a reasonable initial flight speed. A speed is needed to obtain the aerodynamic coefficients. Explore a continous range of values, or iterate several times with the program to compare and converge with the theoretical speed.

Coefficients:

[2] Cl wing lift coefficient --> XFLR5 or CFD
[3] Cle wing extra lift coefficient, set to 1.0 or another justified value
[4] Cd wing drag coefficient --> XFLR5 or CFD
[5] Cde wing extra drag coefficient, set to 1.0 or another justified value. This coefficient makes possible to correct the geometric inaccuracy of the model if does not take into account the billowed cells or the effect of the vents, which undoubtedly increase the drag.
[6] Cm wing moment coefficient --> XFLR5 or CFD. Usually it will be zero, because we will use the resultant forces acting at the center of pressure.
[7] K multiplier coefficient for lift and drag. This value will be an unknown in the equilibrium equations.

Some drags:

[8] Dlines   lines drag (N) --> Automatic calculation made internally by LEP. Calculate the drag of each line taking into account the frontal surface of the line (including loops) and the drag coefficient adapted to the diameter of the line and the Reynolds number Re=ro*V*L/mu. Then drag coefficient according Weisener formula Cdlines=10*Re^(-2/3)+1 and Dragline=(1/2)*ro*Cdl*V^2*Sline. The equivalent diameter of the spliced part is considered D*sqrt(2) and and if the loop is stitched D*2.
[9] Dpilot pilot drag (N) --> Automatic calculation knowing frontal area and aerodynamic drag coefficient. Dragpilot=(1/2)*ro*Cdpilot*V^2*Spilot

Angles:

[10] alpha wing angle of attack (deg). Deduce from the aerodynamic study, as the angle that provides maximum wing glide ratio.
[11] gamma glide angle (deg). This value will be an unknown in the equilibrium equations.
[12] theta assiette inclination of the central chord above or below the horizon (deg). By definition theta= gamma-alpha (view geometric scheme of the longitudinal equilibrium). If the central chord is tilted below the horizon theta > 0, If the central chord is tilted above the horizon theta < 0,
[13] GR glide ratio --> By definition GR=1/(tan(gamma))

Points:

[14] Xcp y-coordinate center of pressure (m) --> XFLR5 or CFD
[15] Zcp z-coordinate center of pressure (m) --> XFLR5 or CFD
[16] C calage, where to place the pilot with respect to the central chord (m or %). This value will be an unknown in the equilibrium equations.
[17] PP plumb point, intersection of vertical line by the pilot, with the central chord (m or %) --> Automatic geometric deduction by LEP known the other main parameters:
Ypp=Ycalage+hlines*tan(theta)
[18] Gy wing mass y-coordinate (m) --> calculate by weighted chords average, automatic calculus by LEP
[19] Gz wing mass z-coordinate (m) --> calculate by weighted chords average, automatic calculus by LEP
[20] Glx y-coordinate of lines mass (m) --> calculate by weighted average, automatic calculus by LEP
[21] Glz z-coordinate of lines mass (m) --> calculate by weighted average, automatic calculus by LEP
[22] GDly y-coordinate of lines mass (m) → Lines drag y application point calculated by weighted average, automatic calculus by LEP
[23] GDlz z-coordinate of lines mass (m) → Lines drag z application point calculated by weighted average, automatic calculus by LEP

Weights:

[24] Mpilot pilot weight (kg) --> Ask the pilot :)
[25] Mw wing weight (kg)   --> Weighting the wing on the scale, excluding risers and lines.
[26] Mlines lines weight (kg) --> Calculated automatically by LEP knowing the weight per linear meter of each type
[27] Mquicklinks (Kg) → Calculated automatically by LEP
[28] Mtotal (kg)   --> Add all the above masses, automatic by LEP. Multiply all masses by 9.8 to convert from Kg to forces in Newtons.

Geometry:

[29] h the distance from pilot to central chord (in a perpendicular line) (m) --> set by definiton, design criteria

Then all the unknowns painted in green are are explicitly specified by the designer or obtained automatically by the program LEP. Then, remain only three main unknowns:

[7] K multiplier coefficient for lift and drag
[11] gamma glide angle (deg)
[16] C calage (m) or %
Let's consider the three equilibrium equations of statics:

(I) SUM(V)=0
f1(gamma,K)=0

(II) SUM(H)=0
f2(gamma,K)=0

(III) SUM(M)=0
f3(gamma, calage)=0

From (I) & (II) → gamma, K
From (III) → calage

Voilà! All unknowns solved theoretically! :)

All the data and results of the calculation of the longitudinal equilibrium calculation are summarized in section 17 of the file lep-out.txt

In this version of LEP-3.20V, consider these results as informative only. It is necessary to experiment with different example models to verify the accuracy of the theoretical calculation's approximation to reality, and make corrections to the calculation if necessary.

Although there are aspects that can be criticized in the methods used, the first numerical results indicate that the theoretical solutions are consistent with reality, and that therefore the results are useful :)

<u>Up to six methods used to solve the problem</u>

During August and September 2022, we have been studying different strategies to solve the non-linear system of equilibrium equations and obtain results close to the real world. Francois programming in Mathcad and Pere in Fortran, with similar or different strategies, until finally finding solutions that match in Mathcad and Fortran, thus confirming that there are no programming errors… :)

Recommended methods are 1 or 6, as explained above. Method 6 has the advantage that the result is fully analytical. Non-linear equations are solved completely explicitly! So theoretically it is the most accurate. Method 1 solves the *gamma* angle numerically but with two loops. In a first loop an approximate solution is found, and in a second loop the result is refined, so it is also very accurate. In addition, method 1 uses a correction coefficient K that tries to homogenize the aerodynamic forces on the wing with those of gravity. Not proven (still a conjecture), but very similar results to method 6. For practical purposes (discounting some decimals) the results of methods 1 and 6 are the same. It is recommended to do a run with both methods to verify the differences.

APENDIX 1: PHYSICAL INTERPRETATION OF EQUILIBRIUM EQUATIONS

"When a paraglider takes off, achieves one equilibrium situation"

This shows us that it is possible to find an unique solution. Which is what we want. Make a physical-mathematical model that allows us to predict the equilibrium situation. We can imagine the set of paraglider and pilot as a rigid solid, and consider all the forces acting on it. This will be our model.

We can consider Newton's second law applied to our model F=ma. Since the paraglider is in equilibrium, the acceleration is zero, i.e. the sum of the resulting forces is zero F=0.

The forces that act on our model are of two types: gravity forces, and aerodynamic forces.

Gravity forces:
- Pilot weight (Wpilot)
- Wing weight (Wwing)
- Lines + risers weight (Wlines)

Aerodynamic forces:
- Wing lift (Lift), normal to trajectory
- Wing drag (Drag), along trajectory
- Wing moment (Moment), torque around an axis perpendicular to the wing plane of symmetry
- Lines drag (Dlines), along trajectory
- Pilot drag (Dpilot), along trajectory

Thus we have a relatively simple system with only 8 forces involved. Gravitational forces depend only on the mass of each element, and are immediate to determine. On the other hand, the aerodynamic forces are not so easy to determine (but possible), and are proportional to the square of the flight speed.

The application of Newton's second law to the particular case of a system in equilibrium, allows us to consider the three classical equations of statics in the plane:

Sum V = 0   (I)
Sum H = 0   (II)
Sum M = 0   (III)

Let's see the meaning of each equation:

Sum V = 0   (I)

For convenience, we will choose the vertical axis in the direction of gravity. This equation tells us that the vertical component of all aerodynamic forces must balance the total weight.

Sum H = 0   (II)

For convenience, we will choose the horizontal axis in the direction perpendicular to that of gravity. This equation tells us that the horizontal component of all the aerodynamic forces cancel out and the sum is zero. Specifically, the horizontal component of Lift is equal to the sum of the horizontal components of the other aerodynamic forces.

Sum M = 0   (III)

For convenience, we choose the reference point for the sum of moments as the center of pressure Cp, as well as the moment caused by the Lift, the Drag, and the moment of the wing itself are zero and do not take part in the equation. We can see how the drag caused by the lines and the pilot produce a negative moment around Cp (counter-clockwise), therefore the moment caused by the total weight around Cp must be positive (clockwise) and of the same magnitude. Realize that the total plumb point must be behind Cp.

We can calculate all the acting forces (gravity and aerodynamics) and their position. And solve the system of the three equations of statics, and so we have the problem solved! :)

Problems encountered while solving the system of equations:

1) The real speed of flight
All aerodynamic forces depend on the speed of flight. To be able to find their values we must first define a speed. That is, we obtain a solution for each speed. But the reality is not that, there is only one flight speed. It is necessary to study different speeds and analyze the results. A simplified formula is proposed in LEP to calculate the theoretical flight speed. However, this formula is under review and some alternative method is being studied to estimate the speed closest to the real one.

2) Model inaccuracy corrections
The values of the coefficient of lift and drag of the wing, obtained by xflr5 and also by CFD are not exact because the geometric model is not accurate enough. In particular the effect of the billow not taken into account in XFLR5 (soon with LEP a model will be prepared in some intermediate profile), and the undetermined effect of the air inlets. We assume that this inaccuracy in the model can be solved by adding an **additional (extra) drag (Dragextra)**. But for the same reason, we can consider that it is necessary to add an **additional lift (Liftextra)**, either positive or negative. To simplify, and in a generic way, we can consider that the coefficients of lift and drag must be affected by coefficients around 1.0 that we will call Clextra, Cdextra. If our CFD model is sufficiently accurate, we can consider Clextra=1.0, Cdextra=1.0.
Lift=(Cl*Clextra)*0.5*ro*V^2*S   Liftextra=(Clextra-1)*0.5*ro*V^2*S
Drag=(Cd*Cdextra)*0.5*ro*V^2*S   Dragextra=(Cdextra-1)*0.5*ro*V^2*S

3) Absolute values of Lift and Drag
When trying to solve the vertical balance, I noticed that in some cases there was no solution. This can happen if the calculated lift is "too big" or "too small", to balance the total weight. But reality shows that there is always balance, no matter how light or heavy the pilot… My interpretation is that the absolute value of the lift (and drag) calculated with Lift=Cl*0.5*ro*V^2*S is not homogeneous with the other calculated drag forces (lines and pilot) and the weights.

This is because to calculate the total lift, a parameter is involved which is the total flat area (or projected) of the wing, which is somewhat arbitrary. To try to make the total lift and drag compatible with the rest of the drags and the total weight, I propose to calculate a reduction or amplification coefficient a coefficient multiplying wing lift and wing drag and solve the equilibrium equations with this applied coefficient.

APENDIX 2: EQUILIBRIUM EQUATIONS

PARAGLIDER LONGITUDINAL EQUILIBRIUM

LIFT

DRAG

CHORD LINE

HORIZONTAL

W_{WING}

GLIDE PATH

DRAG LINES

DL
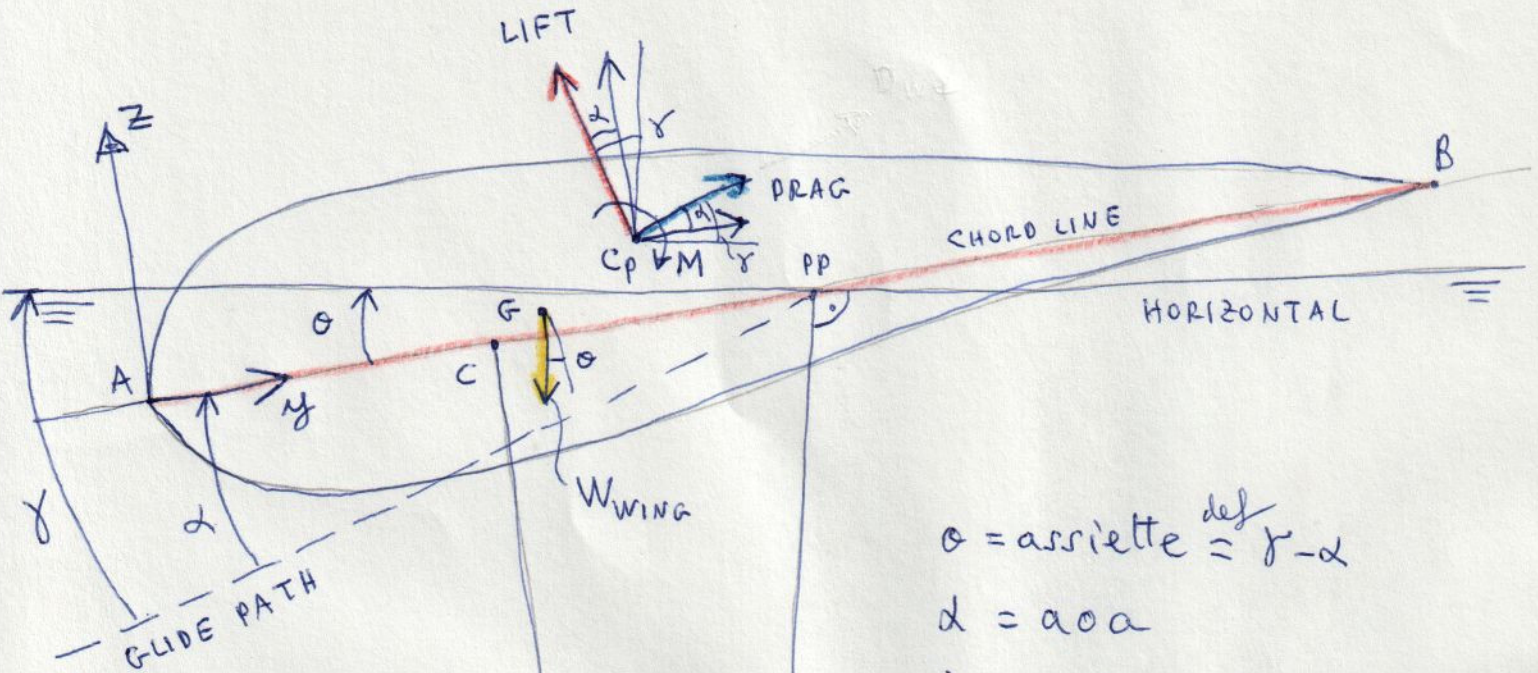
$\theta = assiette \overset{def}{=} \gamma - \alpha$

$\alpha = aoa$

$\gamma = glide\ angle$

$GR = \dfrac{1}{\tan(\gamma)}$

$\overline{AB}$ chord line

$\overline{AC}$ calage

G wing cog

Cp center of pressure

W LINES

GL

PP Plumb point

P pilot position

$\overline{PC} = h$ lines height

EQUILIBRIUM

$$\sum H = 0$$
$$\sum V = 0$$
$$\sum M = 0$$

VERTICAL

h

DRAG PILOT

P
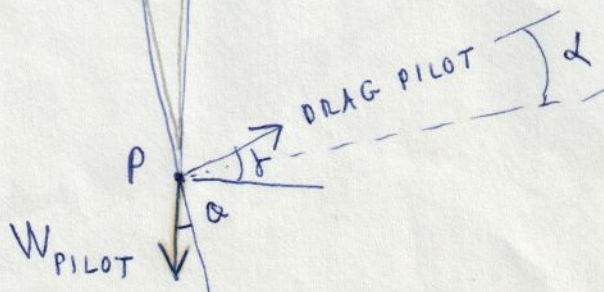
W_{PILOT}

Forces and angles in equilibrium of moments around Cp:
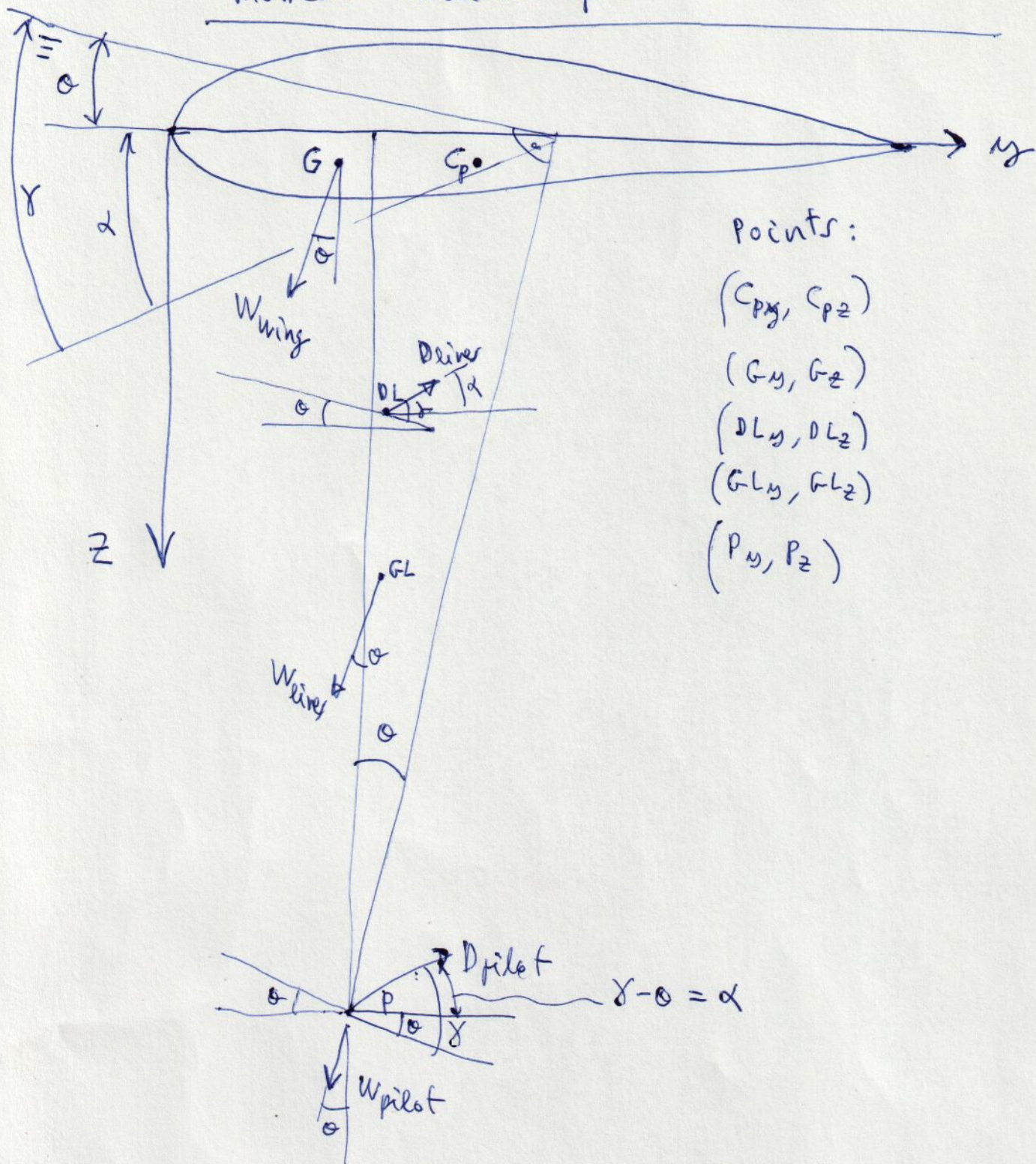
Points:

$$(C_{py}, C_{pz})$$

$$(G_y, G_z)$$

$$(DL_y, DL_z)$$

$$(GL_y, GL_z)$$

$$(P_y, P_z)$$

$$\gamma - \theta = \alpha$$

Generic forces:
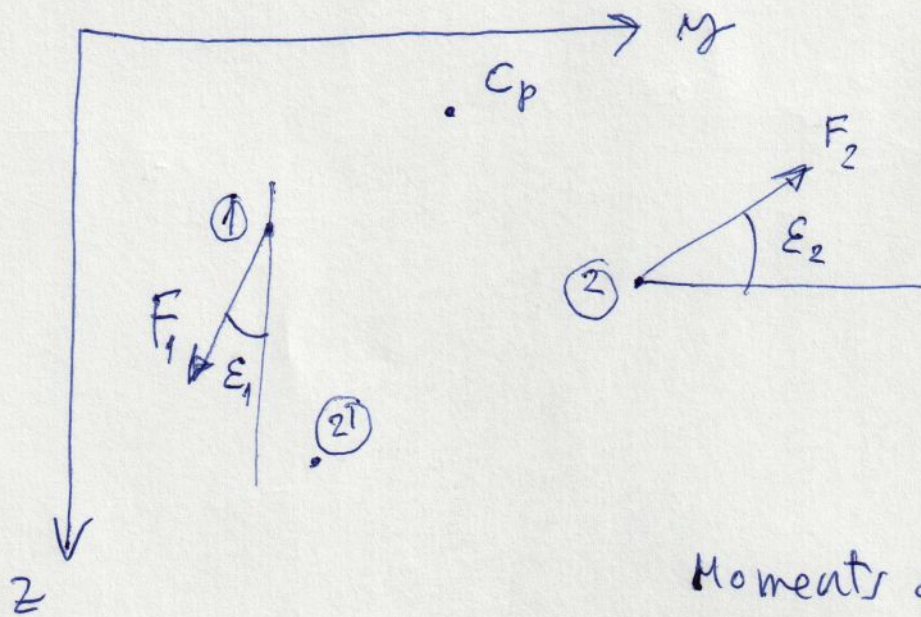
$$+\lambda \quad -\left(C_{py} - 1_y\right) F_1 \cos \varepsilon_1$$

$$= \left(C_{pz} - 1_z\right) F_1 \sin \varepsilon_1$$

$$\left(C_{py} - 2_y\right) F_2 \sin \varepsilon_2$$

$$\left(C_{pz} - 2_z\right) F_2 \cos \varepsilon_2$$

Equilibrium equations

$$+\uparrow \sum V = 0 \quad (I)$$

$$\text{Lift} \cdot \cos\gamma + (\text{Drag} + \text{Dragliner} + \text{Dragpilot}) \cdot \sin\gamma$$

$$- W_{wing} - W_{liner} - W_{ql} - W_{pilot} = 0$$

$$\xrightarrow{+} \sum H = 0 \quad (II)$$

$$-\text{Lift} \cdot \sin\gamma + (\text{Drag} + \text{Dragliner} + \text{Dragpilot}) \cdot \cos\gamma = 0$$

$$\overset{+\curvearrowright}{\sum} M = 0 \quad (III)$$

Moments around C_p:

$$-(C_{py} - G_y) W_{wing} \cdot \cos\theta - (C_{pz} - G_z) W_{wing} \cdot \sin\theta$$

$$-(C_{py} - GL_y) W_{liner} \cdot \cos\theta - (C_{pz} - G_{Lz}) W_{liner} \cdot \sin\theta$$

$$-(C_{py} - P_y) W_{pilot} \cdot \cos\theta - (C_{pz} - P_z) W_{pilot} \cdot \sin\theta$$

$$+(C_{py} - DL_y) D_{liner} \cdot \sin\alpha + (C_{pz} - DL_z) D_{liner} \cdot \cos\alpha$$

$$+(C_{py} - P_y) D_{pilot} \cdot \sin\alpha + (C_{pz} - P_z) D_{pilot} \cdot \cos\alpha = 0$$

moments by vertical components   moments by horizontal components

In equations (I) and (II)

$$\text{Lift} = \frac{1}{2} \cdot \rho \cdot C_\ell \cdot C_{\ell e} \cdot k \cdot V^2 \cdot S$$

$$\text{Drag} = \frac{1}{2} \rho \cdot C_d \cdot C_{de} \cdot k \cdot V^2 \cdot S$$

$$\text{Draglines} = \sum \frac{1}{2} \rho \cdot C_{dlines} \cdot V^2 \cdot S_{line}$$

$$\text{Dragpilot} = \frac{1}{2} \rho \cdot C_{dpilot} \cdot V^2 \cdot S_{pilot}$$

Equation I depends on (k, gamma)

Equation II depends on (k, gamma)

From I $\longrightarrow$ $k_1 = f_1(\text{gamma})$

From II $\longrightarrow$ $k_2 = f_2(\text{gamma})$

For gamma $1°$ to $45°$ .... (normal values)

$k_1 \simeq k_2 \simeq k$? Then

solution is (k, gamma)

$\left.\begin{array}{c} \\ \\ \\ \\ \end{array}\right\}$ Numerical solution

In equation III

(Coordinates using LEP axes Y, Z)

(C_{py}, C_{pz}) center of pressure
(G_y, G_z) Wing center of mass
(P_y, P_z) Pilot position, where $P_y = $ "calage"
(DL_y, DL_z) Lines drag point of application
(GL_y, GL_z) Lines+risers center of mass

$\left.\rule{0pt}{80pt}\right\}$ points

Solve for P_y:

$$P_y = \left(-(C_{py} - G_y)\, W_{wing}\cdot \cos\alpha - (C_{pz} - G_z)\, W_{wing}\cdot \sin\alpha \right.$$

$$- (C_{py} - GL_y)\, W_{lines}\cdot \cos\alpha - (C_{pz} - G_{Lz})\, W_{risers}\cdot \sin\alpha$$

$$- C_{py}\, W_{pilot}\cdot \cos\alpha - (C_{pz} - P_z)\, W_{pilot}\cdot \sin\alpha$$

$$+ (C_{py} - DL_y)\, D_{lines}\cdot \sin\alpha + (C_{pz} - DL_z)\, D_{lines}\cdot \cos\alpha$$

$$+ C_{py}\, D_{pilot}\cdot \sin\alpha + (C_{pz} - P_z)\, D_{pilot}\cdot \cos\alpha \left.\right) \Big/$$

$$\left(D_{pilot}\cdot \sin\alpha - W_{pilot}\cdot \cos\alpha \right)$$

Where $\theta \overset{def}{=} \gamma - \alpha$

Then $P_y \leftarrow$ "calage"

APENDIX 3: POSITION OF THE MASS CENTER OF PILOT+HARNESS

In the system of equations in Appendix 2, we have made an improvement consisting in considering the real position of the pilot, not exactly at the point P (Py,Pz) of the main carabiners, but at a certain distance (named Pmc in data file, oe "s" in the nest figure) below the point P, and in the same vertical (plumb line). Thus equation (III) of appendix 2 must be corrected in the symbols:
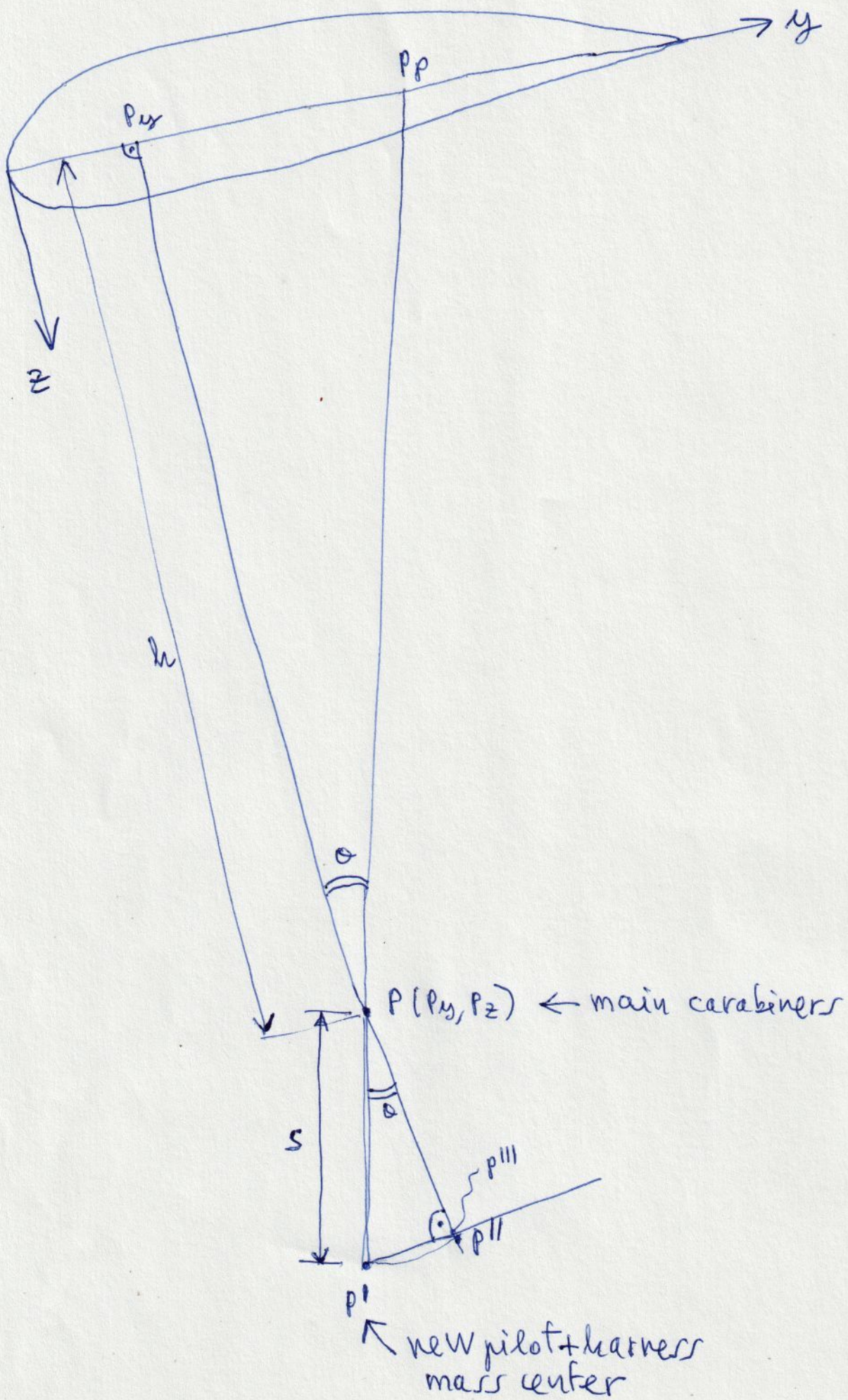
Py by Py-Pmc*sin(theta)
Pz by Pz+Pmc*cos(theta)

This is how we did it in the program, and the results are more accurate. You can verify with the calage, which varies slightly if the pilot's position is not exactly at point P of the carabiners. **Calage is still considered as the y-component of point P**, not the y-component of the pilot+harness mass center (point P'). This is a convention, to be consistent with what is stated in section 8 of leparagliding.txt.

We have drawn the interpretation of the distance that can exist between the center of mass of the pilot+harness assembly with respect to the main carabiners. See figure in next page. The red line PG which is the distance Pmc. In this case we have also drawn an epsilon angle in situations where the center of gravity of the pilot+harness swings forward or backward. But that is another matter of study… :) When studying flight in stable equilibrium, we consider angle epsilon to be zero.

TE

C   PP

LE

A   B   C

MAIN KARABINER → P

G

G pilot + harness
below
main karabiner

ε
rotation

vertical (plumb line)

P_{PY}

P_P

y

z

h

Θ

$P(P_Y, P_Z)$ ← main carabiners

S

Θ

P'''

P''

P'

↖ new pilot+harness
mass center
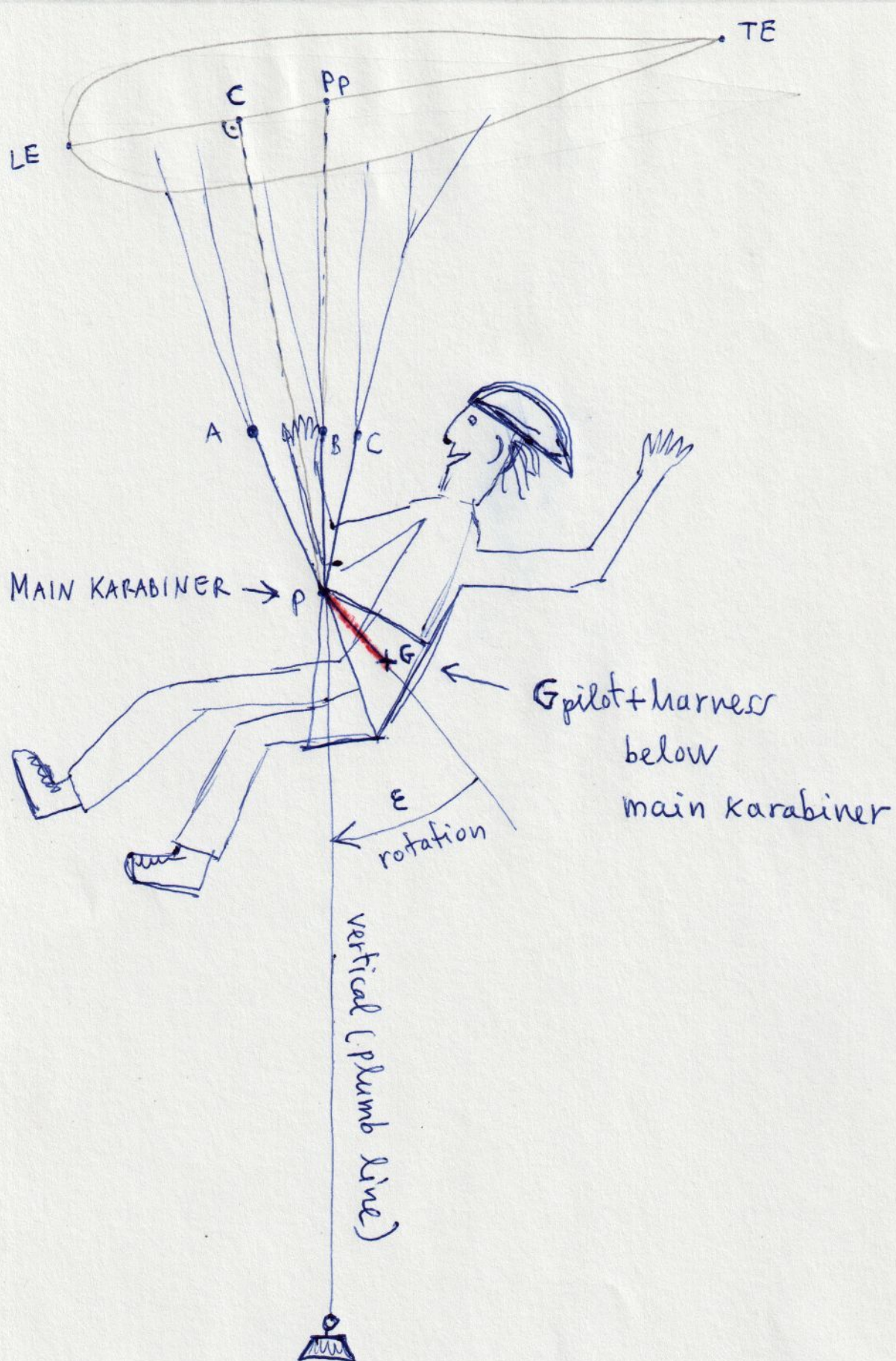
APENDIX 4: SOLVING EQUILIBRIUM EQUATIONS EXPLICITLY USING METHOD 6, AND EXPLAINED IN MATHCAD

Method 6 explained in Mathcad, by Francois de Villiers:
- Solve gamma (glide angle) using horizontal equilibrium
- Solve V (flight speed) using vertical equilibrium
- Solve Py (calage point) using equilibrium of moments

$$L_w = K_l \cdot \frac{1}{2} \cdot \rho \cdot C_l \cdot V^2 \cdot A \quad ....1 \qquad\qquad D_p = cd_p \cdot \frac{1}{2} \cdot \rho \cdot V^2 \cdot A_p \quad .....3$$

$$D_w = K_d \cdot \frac{1}{2} \cdot \rho \cdot C_d \cdot V^2 \cdot A \quad .....2 \qquad\qquad D_l = cd_{line} \cdot \left(\frac{1}{2} \cdot \rho \cdot V^2\right) \cdot A_{line} \quad ......4$$

$$L_w \cdot \cos(\gamma) + \left(D_w + D_l + D_p\right) \cdot \sin(\gamma) - M_{wing} \cdot g - M_{line} \cdot g - M_{ql} \cdot g - M_{pilot} \cdot g = 0 \quad ....5$$

$$-L_w \cdot \sin(q) + \left(D_w + D_l + D_p\right) \cdot \cos(q) = 0 \qquad .....6$$

1 to 4 into 5

$$\frac{V^2 \cdot \rho \cdot \left(A_p \cdot cd_p \cdot \cos(q) + A_{line} \cdot cd_{line} \cdot \cos(q) + A \cdot C_d \cdot K_d \cdot \cos(q) - A \cdot C_l \cdot K_l \cdot \sin(q)\right)}{2} = 0$$

multiply by 2 and divide by V^2 and rho

$$0 = A_p \cdot cd_p \cdot \cos(\gamma) + A_{line} \cdot cd_{line} \cdot \cos(\gamma) + A \cdot C_d \cdot K_d \cdot \cos(\blacksquare) - A \cdot C_l \cdot K_l \cdot \sin(\gamma)$$

so gamma is not a function of velocity !!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!

solve for γ

$$\gamma := 2 \cdot atan\left(\frac{\sqrt{A^2 \cdot C_d^2 \cdot K_d^2 + A^2 \cdot C_l^2 \cdot K_l^2 + 2 \cdot A \cdot A_p \cdot C_d \cdot K_d \cdot cd_p + 2 \cdot A \cdot A_{line} \cdot C_d \cdot K_d \cdot cd_{line} + A_p^2 \cdot cd_p^2 + 2 \cdot A_p \cdot A_{line} \cdot cd_p \cdot cd_{line} + A_{line}^2 \cdot cd_{line}^2} - A \cdot C_l \cdot K_l}{A_p \cdot cd_p + A_{line} \cdot cd_{line} + A \cdot C_d \cdot K_d}\right)$$

from   5

$$L_w \cdot \cos(\gamma) + \left(D_w + D_l + D_p\right) \cdot \sin(\gamma) - M_{wing} \cdot g - M_{line} \cdot g - M_{ql} \cdot g - M_{pilot} \cdot g = 0$$

1 to 4 into 5 and solve for V

$$V := \frac{\sqrt{2} \cdot \sqrt{\rho \cdot g \cdot \left(M_{ql} + M_{line} + M_{wing} + M_{pilot}\right) \cdot \left(A_p \cdot cd_p \cdot \sin(\gamma) + A_{line} \cdot cd_{line} \cdot \sin(\gamma) + A \cdot C_l \cdot K_l \cdot \cos(\gamma) + A \cdot C_d \cdot K_d \cdot \sin(\gamma)\right)}}{2 \cdot \left(\frac{A_p \cdot \rho \cdot cd_p \cdot \sin(\gamma)}{2} + \frac{A_{line} \cdot \rho \cdot cd_{line} \cdot \sin(\gamma)}{2} + \frac{A \cdot C_l \cdot K_l \cdot \rho \cdot \cos(\gamma)}{2} + \frac{A \cdot C_d \cdot K_d \cdot \rho \cdot \sin(\gamma)}{2}\right)}$$

$$g := \frac{g}{\frac{m}{sec^2}}$$

				$cd_{line} \equiv 1.07857$
$C_l := 0.55619$	$A := 12.4577$	$K_l := 1$	$g = 9.807$	$A_{line} := 1 \cdot 0.2515$
$C_d := 0.03560$	$\rho := 1.225$	$K_d := 1.4$		$A_p \equiv 0.4380$
$M_{pilot} := 65.9$				$cd_p := 0.6$

$M_{wing} := 5$

$M_{line} := 0.295$

$M_{riser} := 0.050$ ...riser included in lines

$M_{ql} := 0.048$ 6x8

$M_t := M_{pilot} + M_{wing} + M_{line} + M_{ql} = 71.243$

$$\gamma := 2 \cdot atan\left(\frac{\sqrt{A^2 \cdot C_d^2 \cdot K_d^2 + A^2 \cdot C_l^2 \cdot K_l^2 + 2 \cdot A \cdot A_p \cdot C_d \cdot K_d \cdot cd_p + 2 \cdot A \cdot A_{line} \cdot C_d \cdot K_d \cdot cd_{line} + A_p^2 \cdot cd_p^2 + 2 \cdot A_p \cdot A_{line} \cdot cd_p \cdot cd_{line} + A_{line}^2 \cdot cd_{line}^2} - A \cdot C_l \cdot K_l}{A_p \cdot cd_p + A_{line} \cdot cd_{line} + A \cdot C_d \cdot K_d}\right)$$

$\gamma = 9.463 \cdot deg$

$$V := \frac{\sqrt{2} \cdot \sqrt{\rho \cdot g \cdot (M_{ql} + M_{line} + M_{wing} + M_{pilot}) \cdot (A_p \cdot cd_p \cdot \sin(\gamma) + A_{line} \cdot cd_{line} \cdot \sin(\gamma) + A \cdot C_l \cdot K_l \cdot \cos(\gamma) + A \cdot C_d \cdot K_d \cdot \sin(\gamma))}}{2 \cdot \left(\frac{A_p \cdot \rho \cdot cd_p \cdot \sin(\gamma)}{2} + \frac{A_{line} \cdot \rho \cdot cd_{line} \cdot \sin(\gamma)}{2} + \frac{A \cdot C_l \cdot K_l \cdot \rho \cdot \cos(\gamma)}{2} + \frac{A \cdot C_d \cdot K_d \cdot \rho \cdot \sin(\gamma)}{2} \right)}$$

$V = 12.743$

$W_w := M_{wing} \cdot g \qquad W_l := M_{line} \cdot g$

$W_p := M_{pilot} \cdot g$

$C_{pz} := 0.299 \qquad C_{py} := 0.489$

$G_y := 0.902 \qquad G_z := 0.499$

$GL_y := 1.003 \qquad GL_z := 2.507$

$\alpha := 9.45 \cdot deg$

$P_z := 4.770 + 0.2$

$\theta := (\gamma - \alpha)$

$P_z = 4.97$

$DL_y := 1.0387 \qquad DL_z := 2.1802$

$-(C_{py} - G_y) \cdot W_w \cdot \cos(\theta) - (C_{pz} - G_z) \cdot W_w \cdot \sin(\theta) \; \dots \quad = 0$
$+ -(C_{py} - GL_y) \cdot W_l \cdot \cos(\theta) - (C_{pz} - GL_z) \cdot W_l \cdot \sin(\theta) \; \dots$
$+ -(C_{py} - P_y) \cdot W_p \cdot \cos(\theta) - (C_{pz} - P_z) \cdot W_p \cdot \sin(\theta) \; \dots$
$+ (C_{py} - DL_y) \cdot D_l \cdot \sin(\alpha) + (C_{pz} - DL_z) \cdot D_l \cdot \cos(\alpha) \; \dots$
$+ (C_{py} - P_y) \cdot D_p \cdot \sin(\alpha) + (C_{pz} - P_z) \cdot D_p \cdot \cos(\alpha)$

$PP = P_z \cdot \tan(\theta) + P_y$

$D_p := cd_p \cdot \frac{1}{2} \cdot \rho \cdot V^2 \cdot A_p$

$L_w := K_l \cdot \frac{1}{2} \cdot \rho \cdot C_l \cdot V^2 \cdot A$

$D_l := cd_{line} \cdot \left(\frac{1}{2} \cdot \rho \cdot V^2 \right) \cdot A_{line}$

$D_w := K_d \cdot \frac{1}{2} \cdot \rho \cdot C_d \cdot V^2 \cdot A$

$P_y := -\dfrac{W_w \cdot \cos(\theta) \cdot (C_{py} - G_y) - D_p \cdot \cos(\alpha) \cdot (C_{pz} - P_z) + W_l \cdot \cos(\theta) \cdot (C_{py} - GL_y) + W_w \cdot \sin(\theta) \cdot (C_{pz} - G_z) + W_l \cdot \sin(\theta) \cdot (C_{pz} - GL_z) + W_p \cdot \sin(\theta) \cdot (C_{pz} - P_z) - C_{py} \cdot D_p \cdot s}{D_p \cdot \sin(\alpha) - W_p \cdot \cos(\theta)}$

$P_y = 0.723$

$\dots \mathrm{in}(\alpha) + C_{py} \cdot W_p \cdot \cos(\theta) - D_l \cdot \cos(\alpha) \cdot (C_{pz} - DL_z) - D_l \cdot \sin(\alpha) \cdot (C_{py} - DL_y)$

$PP := (P_z \cdot \tan(\theta) + P_y) \qquad PP = 0.725$

$chord_{center} := 2.121$

$Calage_p := \dfrac{P_y}{chord_{center}} \cdot 100 = 34.107 \qquad PP_p := \dfrac{PP}{chord_{center}} \cdot 100 = 34.162$

$\dfrac{L_w}{D_w} = 11.1595$

$(P_y - C_{py}) \cdot m = 234.41 \cdot mm$

$(PP - C_{py}) \cdot m = 235.579 \cdot mm$

$(PP - P_y) \cdot m = 1.169 \cdot mm$

$\dfrac{L_w}{D_w} = 11.1595$

$\dfrac{L_w}{(D_w + D_p + D_l)} = 5.9993$